Consistency of pole tide and GIA corrections

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Summary

- Wahr et al. [2015]: The GIA secular contribution to the pole tide should be the subtracted mean pole.
- IERS Conventions [2010, updated]: The currently recommended “practical approach” since 2018.
- Polar motion observations update [08/07/24]: Care should be taken with the C01 data.
- Ghelichkhan et al. [2025]: Further clarifications on the origin of the secular polar motion.
- What shall we do now?

Wahr et al. [2015]

- "**Here we assume the secular motion is due solely to GIA** (though see *Mitrovica et al.* [2006] for evidence that it might be difficult for GIA models to explain all the observed trend)."
- "But it is impractical to replace… with such a time convolution, since **that would require observations of $m_1(t)$ and $m_2(t)$ stretching back over centuries to millennia necessary to capture the true GIA polar motion signal**."
- "Instead, we recommend the following approach for removing the GIA pole tide signal from GRACE. First, the GIA contributions to $m_1(t)$ and $m_2(t)$ should not be included… when computing the GRACE pole tide correction. Otherwise, the pole tide correction would include a bogus GIA contribution. Instead, **the GIA contribution to the pole tide should be subtracted from the GRACE C_{21}, S_{21} values, after the other pole tide corrections have been made**. In fact, GIA model predictions of the GIA contribution to C_{21}, S_{21} invariably include the contributions from the GIA pole tide signal (the pole tide in both the solid Earth and oceans), in addition to those due to the direct deformation caused by the removal of the ancient ice loads."

Wahr et al. [2015]

- ”Let $\boldsymbol{C_{21}^{GIA}(t)}$ and $\boldsymbol{S_{21}^{GIA}(t)}$ be **the secular contributions to C_{21} and S_{21} from GIA**. These represent the total GIA contributions, **including the GIA pole tide**. And let $\boldsymbol{m_1^{GIA}(t)}$ **and** $\boldsymbol{m_2^{GIA}(t)}$ be **the secular GIA contributions to polar motion**. Then, the sum of the non-GIA pole tide and the total GIA contributions to C_{21} and S_{21}, is”

$$\begin{pmatrix} C_{21}^{\mathrm{PT+GIA}} \\ S_{21}^{\mathrm{PT+GIA}} \end{pmatrix} = \begin{pmatrix} C_{21}^{\mathrm{GIA}} \\ S_{21}^{\mathrm{GIA}} \end{pmatrix} - 1.333\times10^{-9} \begin{pmatrix} [m_1 - m_1^{\mathrm{GIA}}] + 0.0115\,[m_2 - m_2^{\mathrm{GIA}}] \\ [m_2 - m_2^{\mathrm{GIA}}] - 0.0115\,[m_1 - m_1^{\mathrm{GIA}}] \end{pmatrix} + \begin{pmatrix} -2.1778\times10^{-10}\left([m_1 - m_1^{\mathrm{GIA}}] - 0.01724\,[m_2 - m_2^{\mathrm{GIA}}]\right) \\ -1.7232\times10^{-10}\left([m_2 - m_2^{\mathrm{GIA}}] - 0.03365\,[m_1 - m_1^{\mathrm{GIA}}]\right) \end{pmatrix}$$

- **“The values of $\boldsymbol{C_{21}^{PT+GIA}(t)}$ and $\boldsymbol{S_{21}^{PT+GIA}(t)}$ should be subtracted from the GRACE $\boldsymbol{C_{21}}$, $\boldsymbol{S_{21}}$ values**, to get residual C_{21}, S_{21} values that depend only on present-day surface loading and that are optimal for use in inversions for surface mass.”
- “But published **GIA models often do not explicitly list their predicted values for $\boldsymbol{m_1^{GIA}(t)}$ and $\boldsymbol{m_2^{GIA}(t)}$**. So what should be used for those values?”

$$\begin{cases} x + \dfrac{A_m}{\Omega(C-A)}\dot{y} = -\sqrt{\dfrac{5}{3}}\dfrac{MR_e^2}{C-A}\overline{C}^{*}_{2,1} \\ y - \dfrac{A_m}{\Omega(C-A)}\dot{x} = +\sqrt{\dfrac{5}{3}}\dfrac{MR_e^2}{C-A}\overline{S}^{*}_{2,1}, \end{cases}$$

Couhert (2019)

Wahr et al. [2015]

- "GIA modelers adjust the mantle's viscosity profile and the ice sheet deglaciation history, to produce as good a match as possible to a set of observations. **Often, one of those observations is the observed secular rate of polar motion over the last century. Different GIA models might adopt different values for the observed rate... Since we are assuming here that the 1900–1978 polar motion trend is caused entirely by GIA, we propose to skip the GIA modeling process when adopting values for $m_1^{GIA}(t)$ and $m_2^{GIA}(t)$ and, instead, to adopt the observed secular rates of $m_1(t)$ and $m_2(t)$ directly, with the rationale that an optimally functioning GIA model would reproduce those rates. For those rates, we suggest adopting the ILS results of Argus and Gross [2004] for the polar motion drift** in the mean lithosphere frame (the frame that most nearly corresponds to the IERS frame):"

$$\dot{m}_1^{\mathrm{GIA}} = 0.62 \times 10^{-3}\ \mathrm{arcsec/yr}$$

$$\dot{m}_2^{\mathrm{GIA}} = -3.48 \times 10^{-3}\ \mathrm{arcsec/yr}$$

Wahr et al. [2015]

- “Since the data used to determine these rates stopped in the late 1970s, **their trend estimates should not be significantly affected by the recent changes in polar ice sheet mass** discussed in the following subsection. If we use these rates… and combine the last two pole tide terms on the right-hand side into a single term, we obtain a combined pole tide + GIA correction of”

$$\begin{pmatrix} C_{21}^{\mathrm{PT+GIA}}(t) \\ S_{21}^{\mathrm{PT+GIA}}(t) \end{pmatrix} = \begin{pmatrix} C_{21}^{\mathrm{GIA}}(t) \\ S_{21}^{\mathrm{GIA}}(t) \end{pmatrix} + \begin{pmatrix} -1.551\times10^{-9} \\ 0.021\times10^{-9} \end{pmatrix} \left[m_1(t) - 0.62\times10^{-3}(t - t_0)\right]$$

$$+ \begin{pmatrix} -0.012\times10^{-9} \\ -1.505\times10^{-9} \end{pmatrix} \left[m_2(t) + 3.48\times10^{-3}(t - t_0)\right]$$

IERS Conventions [2010, updated]

- "…it is convenient to use a conventional representation of the low-frequency motion of the Earth's rotation axis with respect to the terrestrial reference system free of the principal high-frequency periodic motions. This representation is provided by a linear representation called the **secular pole derived from a least-squares fit to the polar motion observations from 1900 through 2017**. The coordinates of that secular pole designated (x_s, y_s) and are given in milliarcseconds by

$$x_s = 55.0 + 1.677 * (t - 2000), \quad y_s = 320.5 + 3.460 * (t - 2000)$$

⇒No consistency with the recommendation of Wahr et al. [2015] (with possible RIM contaminations).

Polar motion observations update [08/07/24]

- “Full update of CO1 data: The pole coordinates are in better agreement with the Conventional International Origin (CIO) over the period 1900-1906. The former version presented a spurious shift over the interval 1900 - 1962, which has been removed subsequently.”

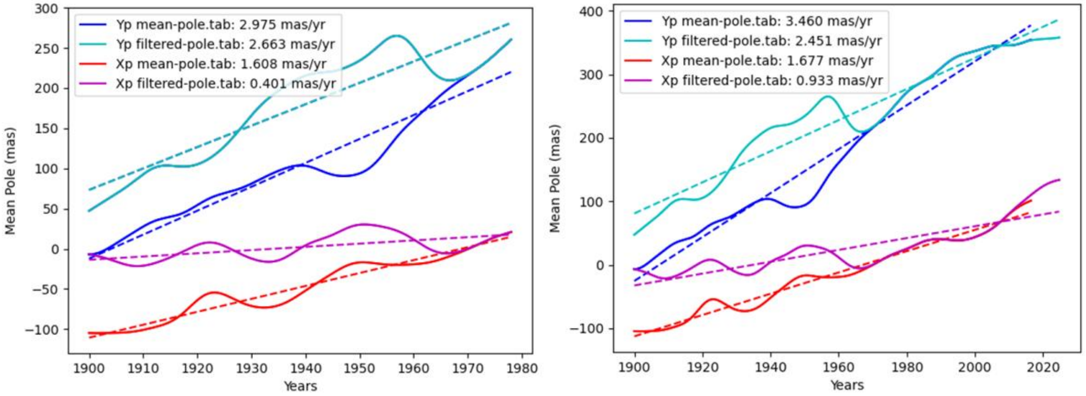


FIGURE : Smoothed IERS polar motion values with their secular rates between 1900–1978 (left) & 1900–2020 (right, current Conventions), before and after the update of C01 data over 1900–1962.

Polar motion observations update [08/07/24]

- “This follows the consideration of the optical astrometry series from the Astronomical Institute of the Czech Academy of Sciences (AICAS), based on the reprocessing of observations compiled from 1900 to 1992 with the positions of stars given by the Hipparcos catalog.”
- “It is important to understand that the drift, discovered by the American astronomer in the 1920s, is uncertain and is affected by the instrumental biases of the time. The proof of this is that the reality of this drift was doubted until the 1960s.”
- “You will notice that in 1900, the mean pole, in all cases, did not have the coordinates (0,0) to within 30 mas—the error at the time—i.e., it did not coincide with the geographic pole. However, by the very definition of the geographic pole since the beginning of the 20th century—when it became clear that it could no longer be confused with the rotational pole—these two poles should be one and the same for the period 1900-1906. In short, we have lost the pole.”
- “**It is therefore best… to attribute… these discrepancies to the uncertainty affecting optical observations.** In principle, the latest AICAS series (2010) is the final arbiter, which is why I have adopted it for the period 1900 to 1962.”

Bizouard (2024)

Ghelichkhan et al. [2025]

- “We conclude that the signal of ongoing **GIA dominates the budget of modern TPW and that mantle convection-induced TPW contributed 25–30 per cent of the observed rate**.”
- “… **we interpret our estimate of the convection signal in TPW to be an upper bound**. Indeed, a reduction of this signal by a factor of two, consistent with long-term rates of TPW (Greff-Lefftz 2004 ; Rouby *et al.* 2010 ; Chan *et al.* 2011), can be accommodated while preserving a fit to the observed 20th century rate.”

What shall we do now?

In the short term:

- **Extract the secular pole from the GIA model to be subtracted in the pole tide correction for consistency.**
- Let $C_{21}^{GIA}(t)$ and $S_{21}^{GIA}(t)$ be the secular contributions to C_{21} and S_{21} from GIA (including the GIA pole tide). Let $m_1^{GIA}(t)$ and $m_2^{GIA}(t)$ be the secular GIA contributions to polar motion.
- Then the total $C_{21}^{PT+GIA}(t)$ and $S_{21}^{GPT+GIA}(t)$ corrections (which include the GIA and the pole tide corrections) can be decomposed into (m_1 and m_2 represent polar motion) the following equation:

$$\begin{pmatrix} C_{21}^{\mathrm{PT+GIA}} \\ S_{21}^{\mathrm{PT+GIA}} \end{pmatrix} = \begin{pmatrix} C_{21}^{\mathrm{GIA}} \\ S_{21}^{\mathrm{GIA}} \end{pmatrix} - 1.333\times10^{-9} \begin{pmatrix} \left[m_1 - m_1^{\mathrm{GIA}}\right] + 0.0115\left[m_2 - m_2^{\mathrm{GIA}}\right] \\ \left[m_2 - m_2^{\mathrm{GIA}}\right] - 0.0115\left[m_1 - m_1^{\mathrm{GIA}}\right] \end{pmatrix} + \begin{pmatrix} -2.1778\times10^{-10}\left(\left[m_1 - m_1^{\mathrm{GIA}}\right] - 0.01724\left[m_2 - m_2^{\mathrm{GIA}}\right]\right) \\ -1.7232\times10^{-10}\left(\left[m_2 - m_2^{\mathrm{GIA}}\right] - 0.03365\left[m_1 - m_1^{\mathrm{GIA}}\right]\right) \end{pmatrix}$$

What shall we do now?

Here are the steps to follow:

- Ocean mass trends derived from satellite gravimetry:
  1) Given the uncertainty in polar motion observations of the secular evolution and the fact that GIA dominates (~75%) the observed rate,
  2) One should extract the time derivative of $m_1^{GIA}(t)$ and $m_2^{GIA}(t)$ from the GIA model (GIA induced rate of polar motion),
  3) And use these estimates to compute the pole tide correction (i.e., the second term in the previous equation). More explicitly, this correction should be subtracted from the GRACE-derived C_{21} and S_{21} values (as well as the usual $C_{21}^{GIA}(t)/S_{21}^{GIA}(t)$):

$$\begin{pmatrix} \Delta \mathrm{C}_{21}^{PT\_COR}(t) \\ \Delta \mathrm{S}_{21}^{PT\_COR}(t) \end{pmatrix} = \begin{pmatrix} -1.551 \times 10^{-9} \\ 0.021 \times 10^{-9} \end{pmatrix} \left[x_s(t) - \dot{m}_1^{GIA}(t-2000) \right] + \begin{pmatrix} -0.012 \times 10^{-9} \\ -1.505 \times 10^{-9} \end{pmatrix} \left[y_s(t) - \dot{m}_2^{GIA}(t-2000) \right]$$

$$x_s(t) = 0.055 + 0.001677 \times (t-2000), y_s(t) = 0.3205 + 0.00346 \times (t-2000)$$

where *t* is the date in years and the GIA induced rate of polar motion are in arcseconds/yr.

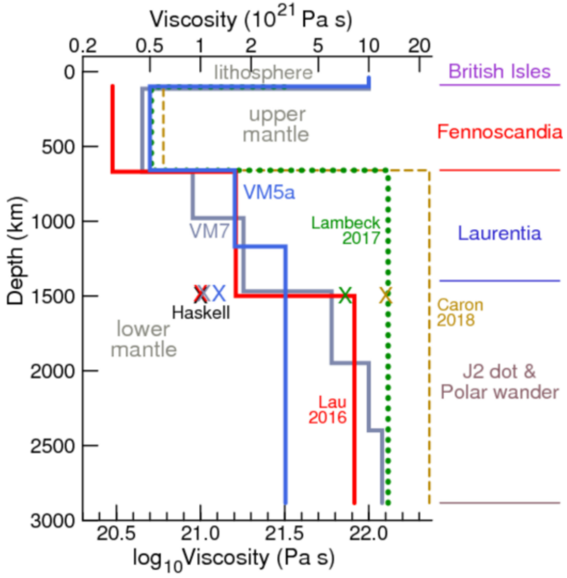
What shall we do now?

Here are the steps to follow:

- Satellite altimetry sea level rise estimates:
  1) Remove the same ‘mean pole’ (secular GIA contributions to polar motion in arcseconds) in the pole tide correction for the satellite altimeter sea surface height (Desai et al., 2015), in place of the 2017 coordinates of the secular pole ($x_s(t)$, $y_s(t)$) recalled in the previous slide.

What shall we do in the future?

- <u>In the foreseeable future:</u> Rousselet et al. [2026]
  - Not use constant Love numbers for all frequency ranges (from sub-daily elastic to centennial transient solid Earth response) to determine the pole tide correction.
  - Same as previous point for the secular pole tide correction.



Argus et al. (2021)