ETH zürich

SPACE GEODESY

Correlations in Atmospheric Parameter Estimation

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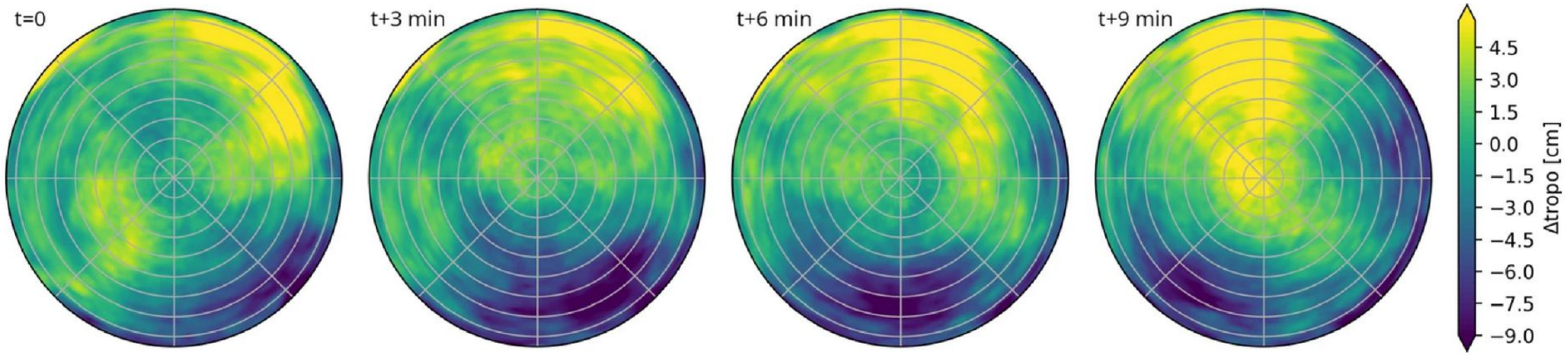
6 March 2026, Unified Analysis Workshop

Correlation of tropospheric effects



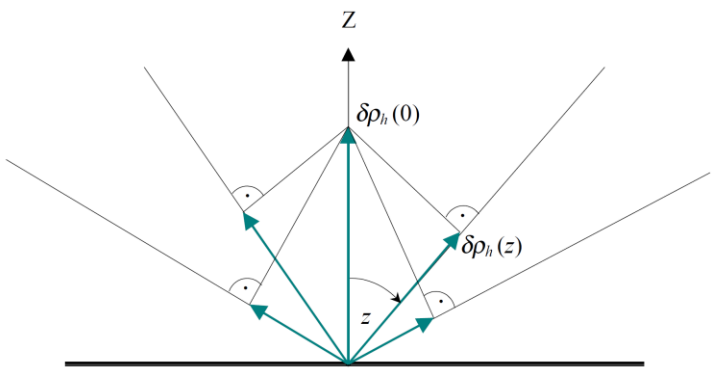
Schartner, 2025

Problem overview

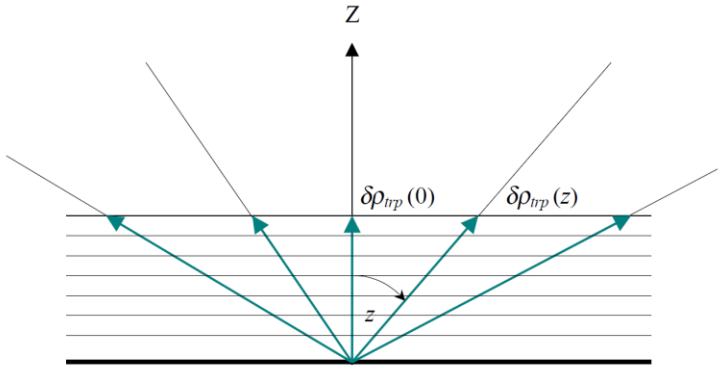
- **Current practice:** most Least Squares Adjustments (LSA) use **diagonal covariance matrices**
  - Assumption: observations are independent
- **Reality:** atmosphere is governed by **Kolmogorov turbulence**
  - Temporal correlations: short-term fluctuations are linked
  - Spatial correlations: co-located stations share the same troposphere
- **Consequence:**
  - Optimistic uncertainties
  - Sub-par tropospheric estimates → errors in station heights

Correlation of station height, atmosphere & clock parameters

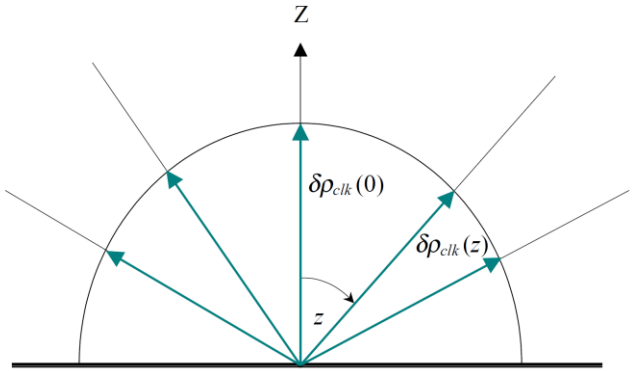
- Partial derivatives



$$\delta\rho_h(z) = \cos(z) \cdot \delta\rho_h(0) \quad ; \quad \delta\rho_h(0) = \delta h$$



$$\delta\rho_{trp}(z) = \frac{1}{\cos(z)} \cdot \delta\rho_{trp}(0)$$



$$\delta\rho_{clk}(z) = \delta\rho_{clk}(0) = c \cdot \delta t_R$$

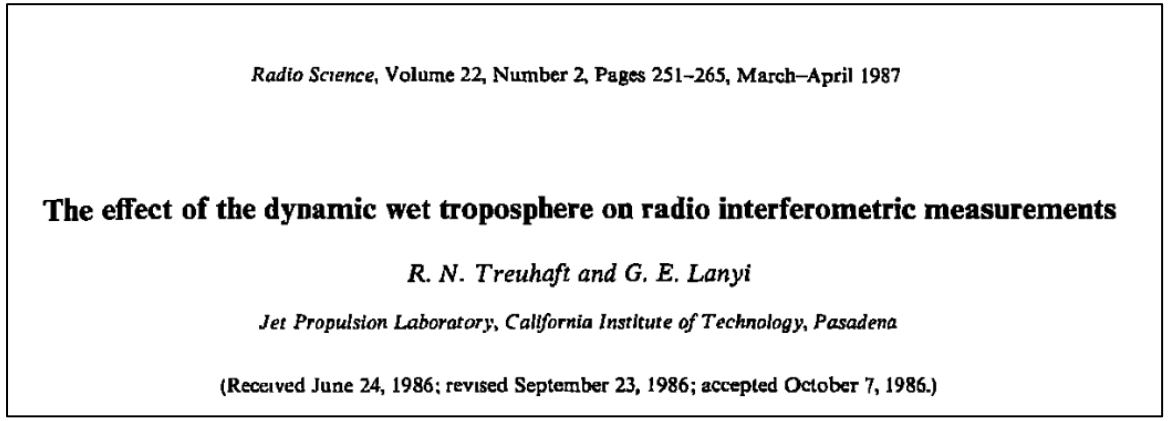
- Errors in atmospheric effects propagate to other geodetic parameters

Atmospheric turbulence

- Tropospheric delays are not random noise but rather **power-law noise** in space and time
  → correlations of observations
- Structure function dependent on distance *r*
- Typical assumption: frozen flow with a certain wind vector

$$D_n(r) = \langle [n(x) - n(x+r)]^2 \rangle = C_n^2 r^{2/3}$$

- **Requires appropriate stochastic modeling**



Radio Science, Volume 22, Number 2, Pages 251–265, March–April 1987

The effect of the dynamic wet troposphere on radio interferometric measurements

R. N. Treuhaft and G. E. Lanyi

Jet Propulsion Laboratory, California Institute of Technology, Pasadena

(Received June 24, 1986; revised September 23, 1986; accepted October 7, 1986.)

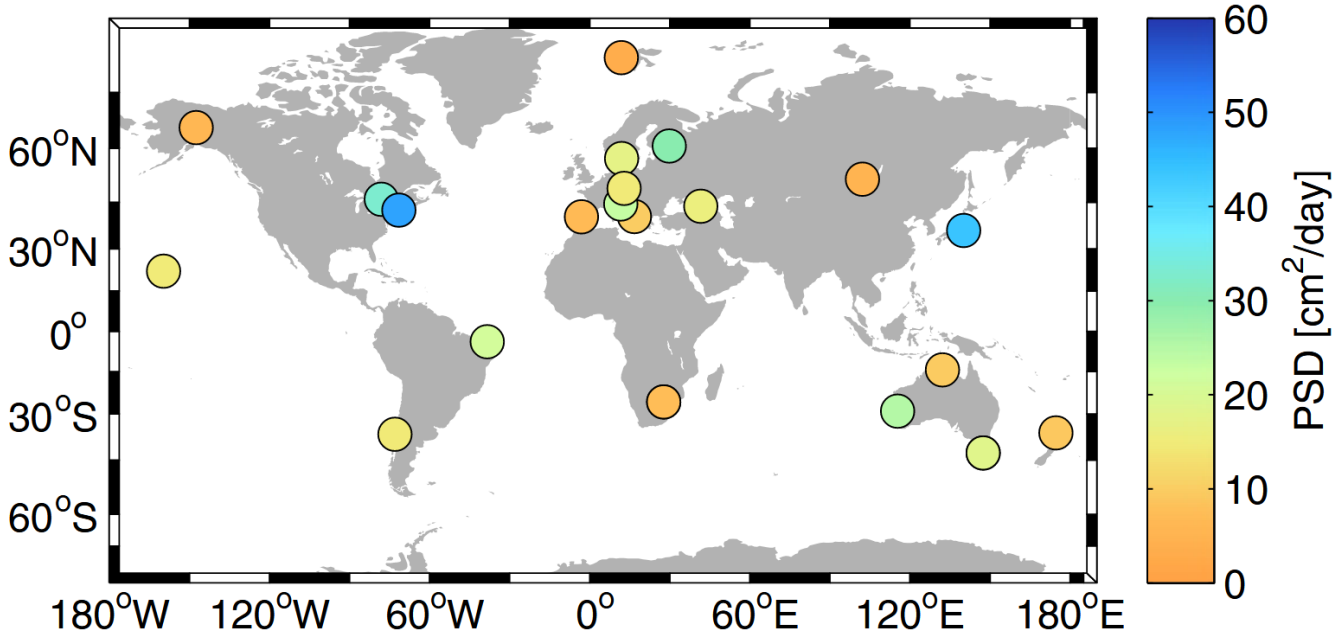
Considering correlations in geodetic parameter estimation

- Ideally: correlations as off-diagonal elements in the weight matrix

- Other concepts:
  - Stochastic modeling in the parameter domain
    - Kalman filter with random walk
    - LSA with piece-wise linear functions and constraints
  - Combination of tropospheric parameters of co-located stations

Station-dependent stochastic parameters

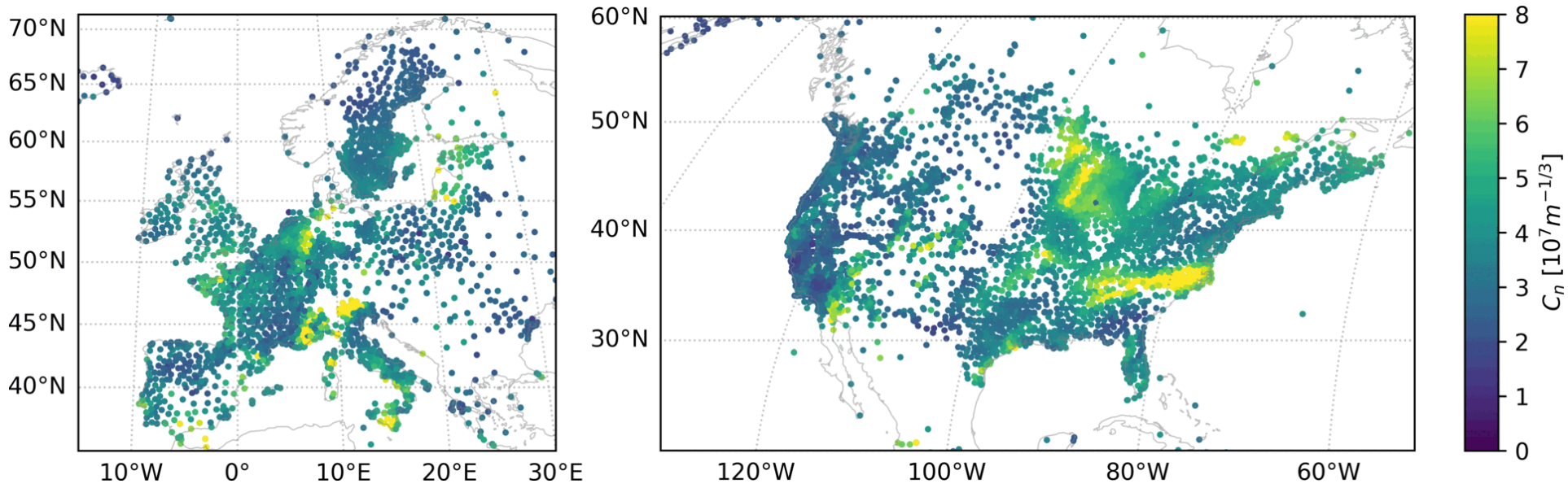
- Stochastic parameters are often assumed to be identical for all stations
- Atmospheric turbulence is actually very inhomogeneous
  - Summer vs. winter
  - Tropical vs. arid regions



Soja et al., 2015

Modeling turbulence parameters

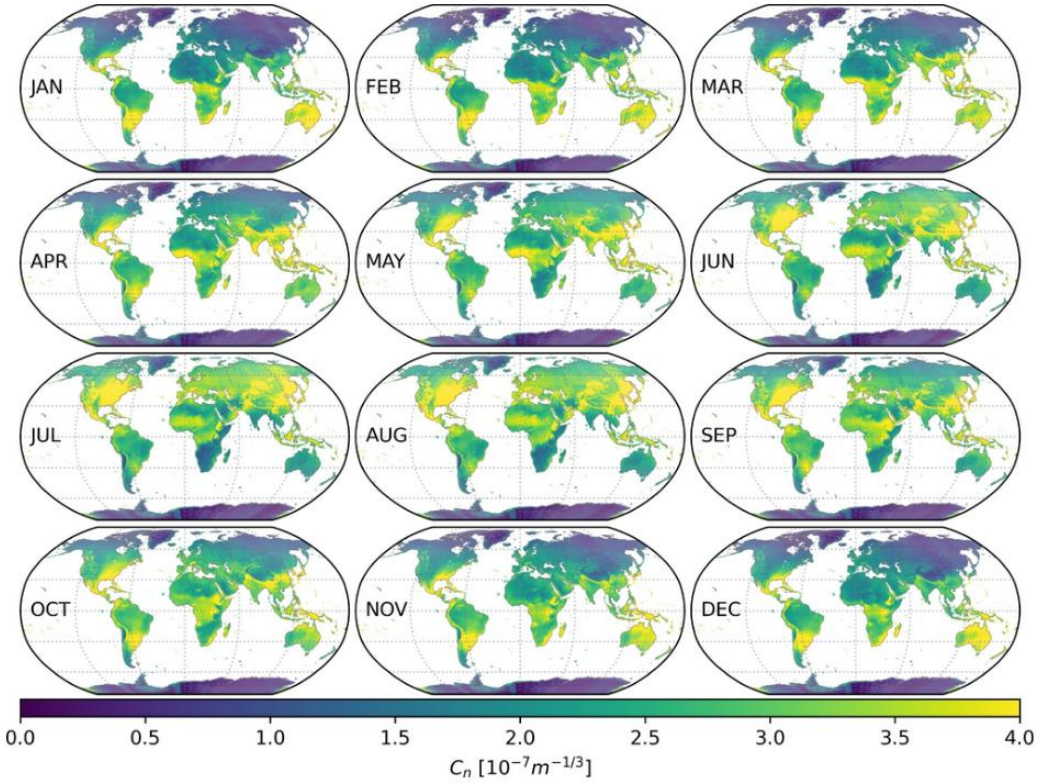
- Machine learning to model Cn values using 18k GNSS stations + numerical weather model



Schartner, 2025

Modeling turbulence parameters

- Full model depends on lat, lon, time
- Easy access: monthly averaged grids



https://gpc.ethz.ch/Troposphere/

Discussion

- Short presentations
  - Hana Krasna
  - Minghui Xu

- Discussion points
  - How are correlations due to the atmosphere handled by the different ACs or services?
  - Impact on geodetic products like ITRF and EOP
  - Should we mention atmospheric turbulence in the IERS Conventions?
  - Are ACs using tropospheric ties at co-located stations?
  - Interest in a multi-technique combined tropospheric product?

Thanks for your attention!

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