Stochastic model in VLBI work done at TU Wien

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Modification of the stochastic model of VLBI observations

Modification of the stochastic model of VLBI observations

- Constant extra noise (33 ps ≈ 1 cm)
- Elevation dependent noise – diagonal matrix (default option at VIE AC)

$$\sigma_{obs}^2 = \sigma_0^2 + \left(\frac{x[\mathrm{ps}]}{\sin(el_1)}\right)^2 + \left(\frac{x[\mathrm{ps}]}{\sin(el_2)}\right)^2$$

- Elevation dependent noise with correlations in a scan

- Turbulence model developed by Halsig (2018) based on Kermarrec and Schön (2014) model: Matérn covariance function (Matérn 1960)

Correlations within a scan

$\left(\frac{x\,[\mathrm{ps}]}{\sin(el_i)}\right)$

Correlated elevation dependent noise

Simplifying assumption
- consideration of **observations at a common time with a station in common**
- the **covariance matrix is block diagonal**

$\Omega_{scan} = \Omega_{meas} + \Omega_A$

Ω_{meas} : diag(σ_{0i}^2) ; $\sigma_i = \frac{x\,[\mathrm{ps}]}{\sin(el_i)}$

One block of a block-diagonal matrix Ω_A (containing elevation dependent noise terms) which includes **one scan consisting of six observations**:

Diagonal terms: baseline ij

$$\Omega_{A,ij,ij} = \sigma_i^2 + \sigma_j^2$$

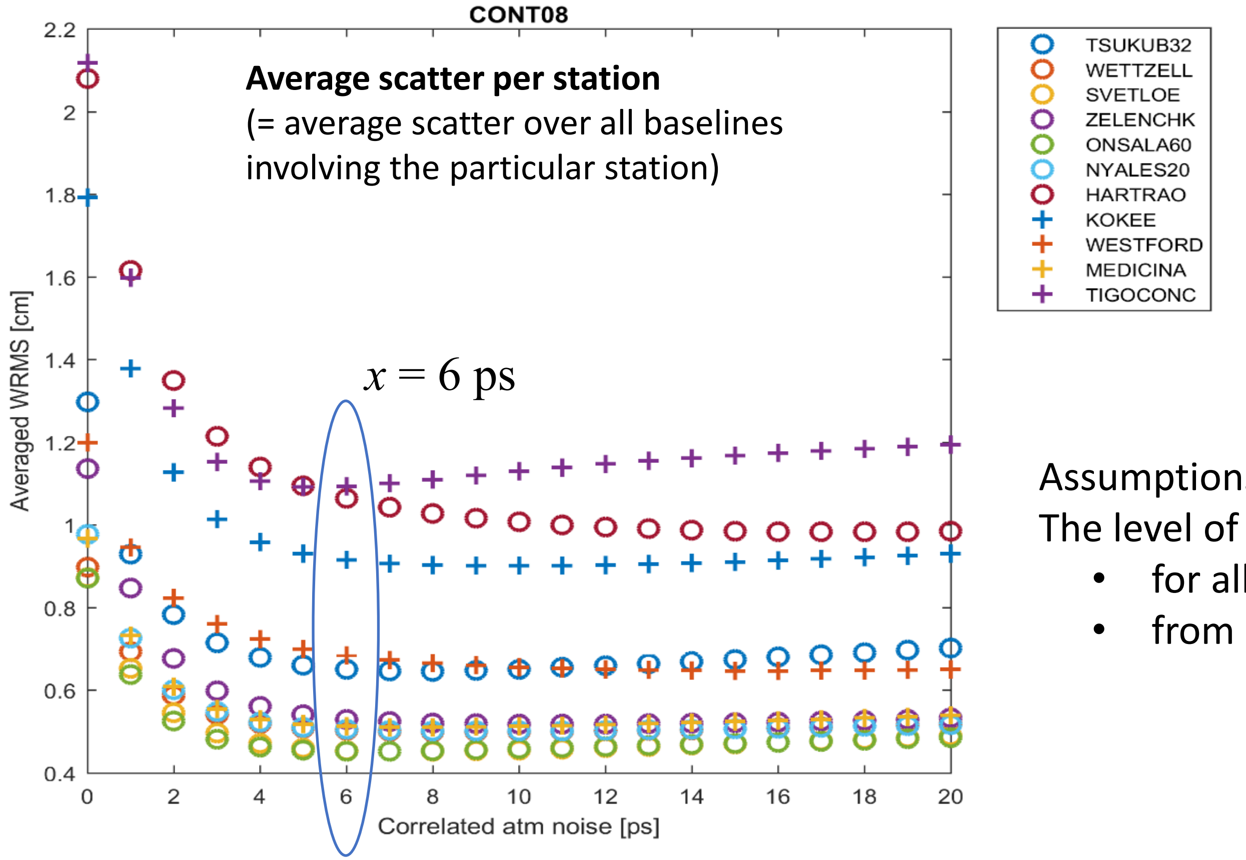
Off-diagonal terms: non-zero only if the baselines have a station in common

$$\Omega_{A,ij,il} = -\Omega_{A,ij,li} = \sigma_i^2$$

Baseline	1-3	1-4	3-4	5-4	1-5	3-5
1-3	$\sigma_1^2+\sigma_3^2$	σ_1^2	$-\sigma_3^2$	0	σ_1^2	$-\sigma_3^2$
1-4	σ_1^2	$\sigma_1^2+\sigma_4^2$	σ_4^2	σ_4^2	σ_1^2	0
3-4	$-\sigma_3^2$	σ_4^2	$\sigma_3^2+\sigma_4^2$	σ_4^2	0	σ_3^2
5-4	0	σ_4^2	σ_4^2	$\sigma_5^2+\sigma_4^2$	$-\sigma_5^2$	$-\sigma_5^2$
1-5	σ_1^2	σ_1^2	0	$-\sigma_5^2$	$\sigma_1^2+\sigma_5^2$	σ_5^2
3-5	$-\sigma_3^2$	0	σ_3^2	$-\sigma_5^2$	σ_5^2	$\sigma_3^2+\sigma_5^2$

Correlated elevation dependent noise

x: 0 – 20 ps



$$\left(\frac{x\ [\mathrm{ps}]}{\sin(el_i)}\right)$$

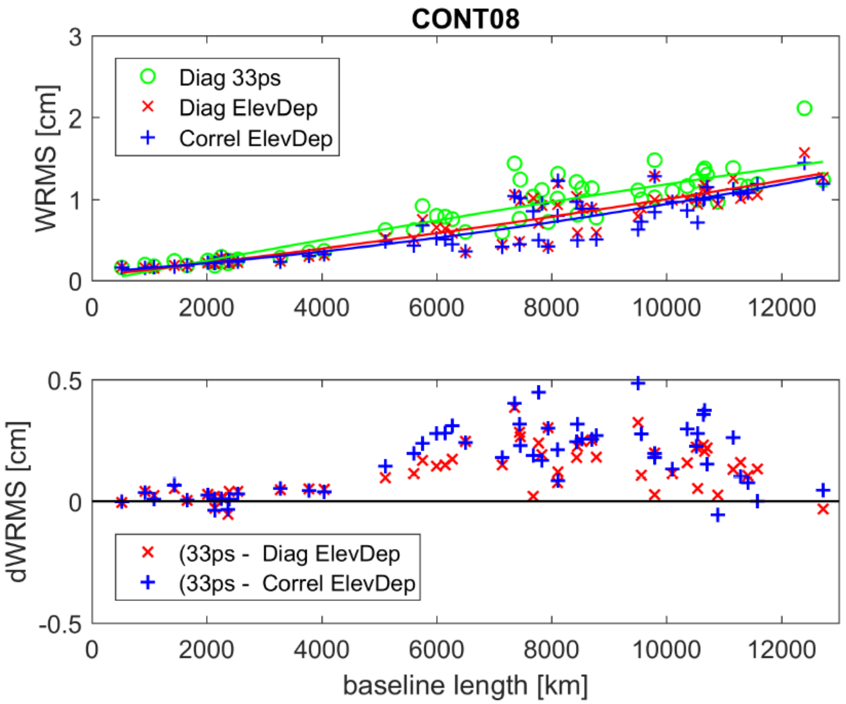
[cm]	ZWD over CONT08 Mean	WRMS
Tsukub32	29.0	5.9
Wettzell	13.4	3.1
Svetloe	14.8	4.2
Zelenchk	16.0	2.2
Onsala60	13.9	2.7
Nyales20	8.1	1.9
Hartrao	7.0	2.7
Kokee	9.1	2.7
Westford	15.3	4.2
Medicina	16.6	3.6
Tigoconc	8.5	4.1

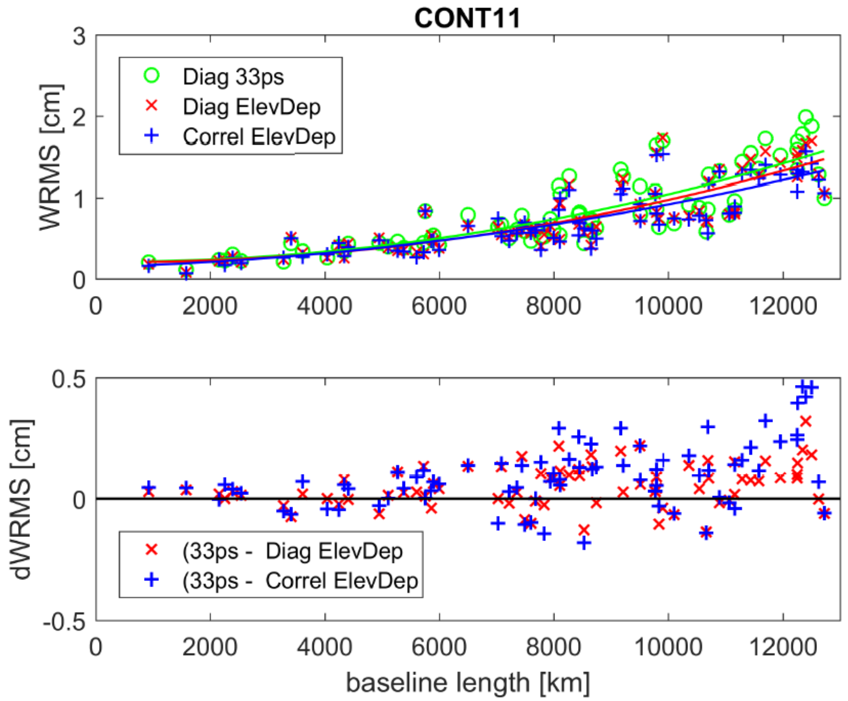
Assumptions for the correlated noise:
The level of noise is the same
- for all stations
- from day to day

- Noisier stations in terms of the baseline length scatter: TIGOCONC, HARTRAO, KOKEE, TSUKUB32

- Dependence of the baseline length scatter on the wet atmospheric delay is not proven

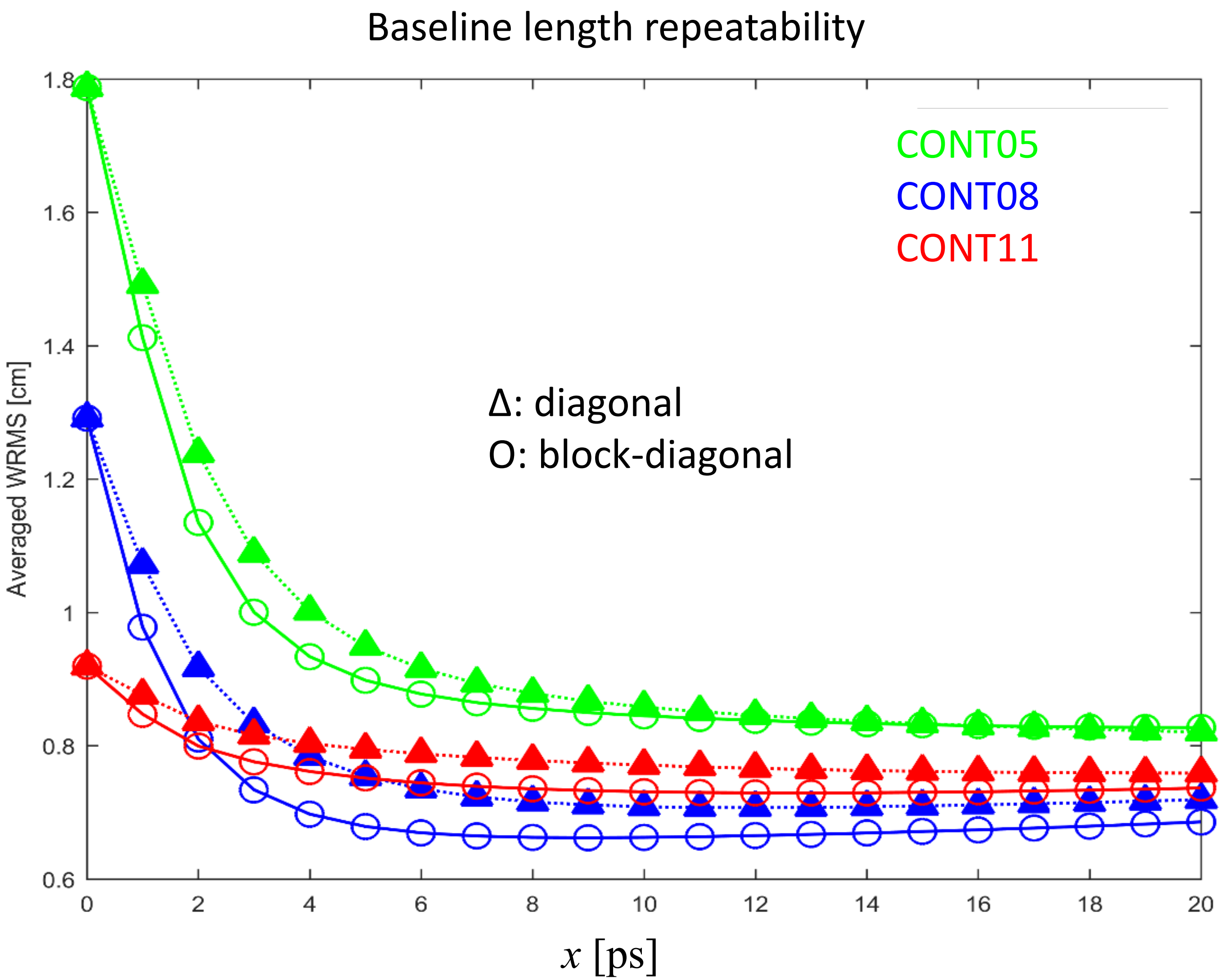
Elevation dependent noise: BLR





	Diag ElevDep		Correl ElevDep	
	Mean improvement [mm]	% of Improved baselines	Mean improvement [mm]	% of Improved baselines
CONT05	1.0	81.8%	1.3	81.8%
CONT08	1.4	92.7%	1.8	92.7%
CONT11	0.5	71.8%	0.9	76.9%

Elevation dependent noise: diagonal x block-diagonal



$$\left(\frac{x\ [\mathrm{ps}]}{\sin(el_i)}\right)$$

Introducing **correlations** between observations **in a scan** **reduces** the **WRMS** of baseline length compared to just a diagonal elevation dependent noise.

Turbulence model by Halsig (2018)

- Turbulence model developed by Halsig (2018) based on Kermarrec and Schön (2014) model:

Matérn covariance function (Matérn 1960)

Variance terms (diagonal):

$$\boldsymbol{C}_{\varphi}(t,t) = 0.782 \frac{k^2 H C_n^2 c \kappa_0^{-\frac{3}{5}}}{\left[mf\left(\varepsilon_i(t)\right)\right]^2}$$

x ps

$\sin^2(\mathrm{el})$

+ Covariance terms (off-diagonal)

Parameters describing the turbulence

H: tropospheric height

C_n^2: refractive index structure constant

c: scaling parameter in the vertical direction

κ_0 : $2\mathrm{pi}/L_0$, L_0 initial size of the eddies

k: signal wavelength

Started PhD thesis, Peter Urban

Implementation in VieVS, investigations on least squares collocation approach to model the stochastic properties, investigations on availability of station-specific turbulence parameters